Time Scale for Rapid Draining of a Supraglacial Lake into the Greenland Ice Sheet

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What are supraglacial lakes?

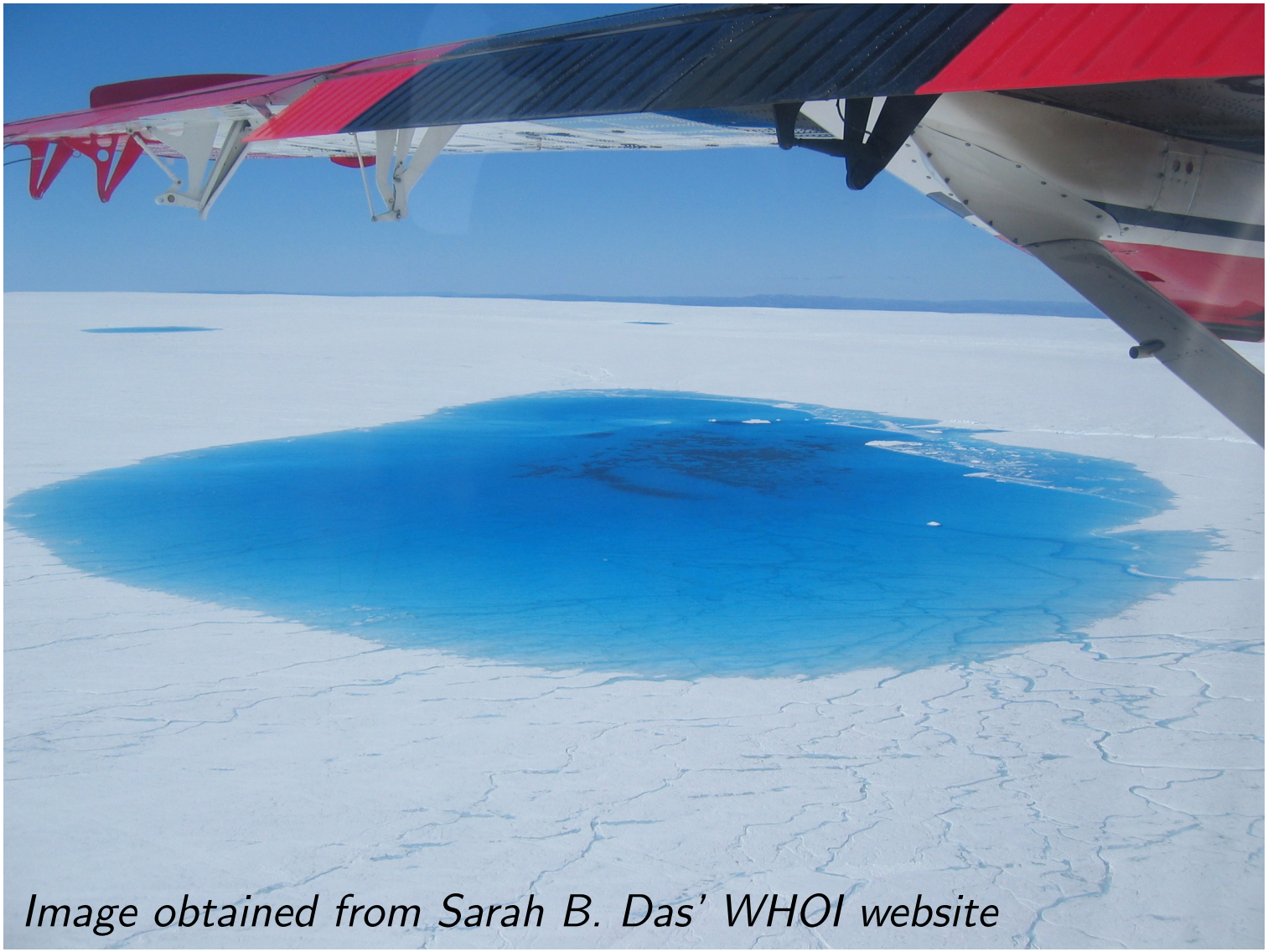


Image obtained from Sarah B. Das' WHOI website

What are supraglacial lakes?

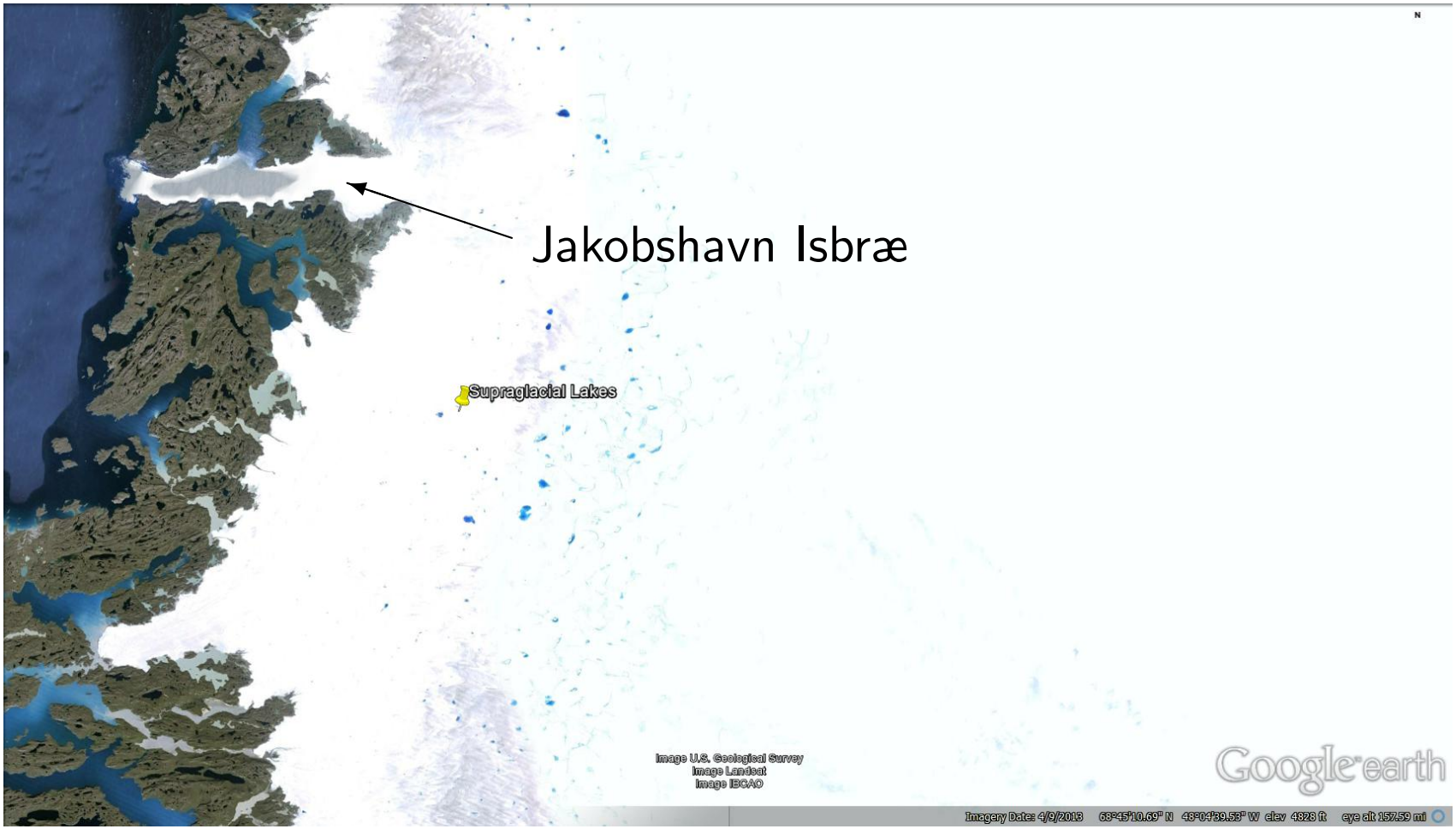
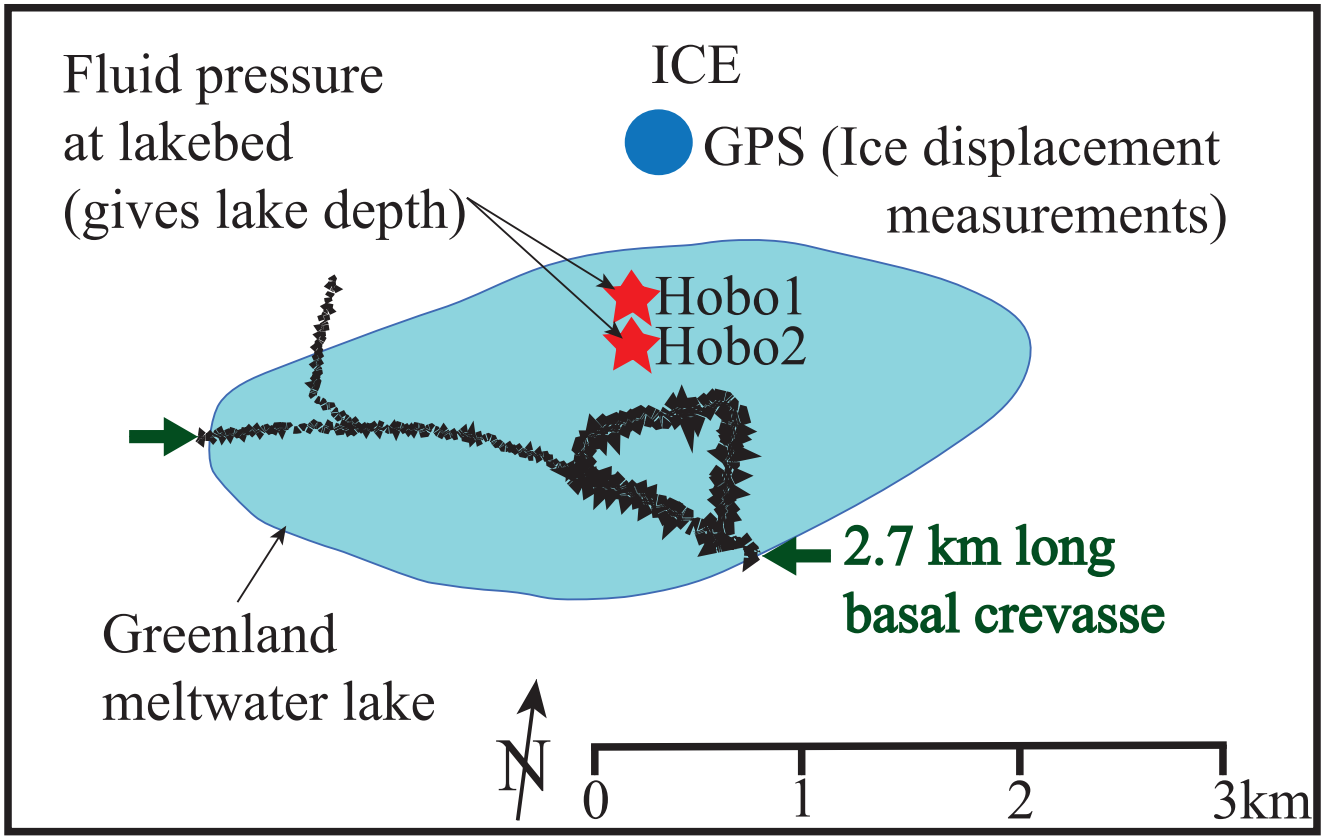
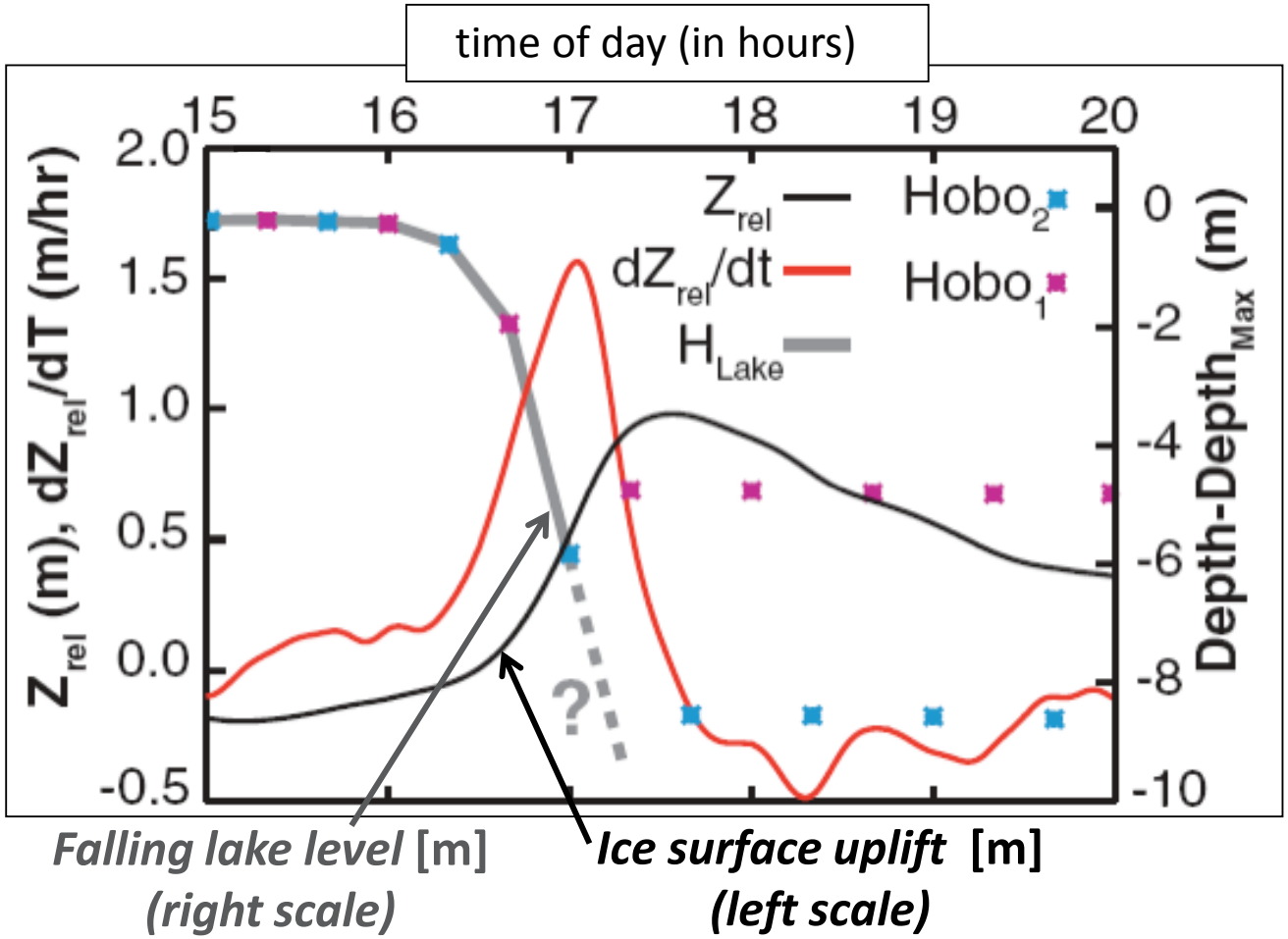


Illustration of data aquisition system by Das et al. (2008)



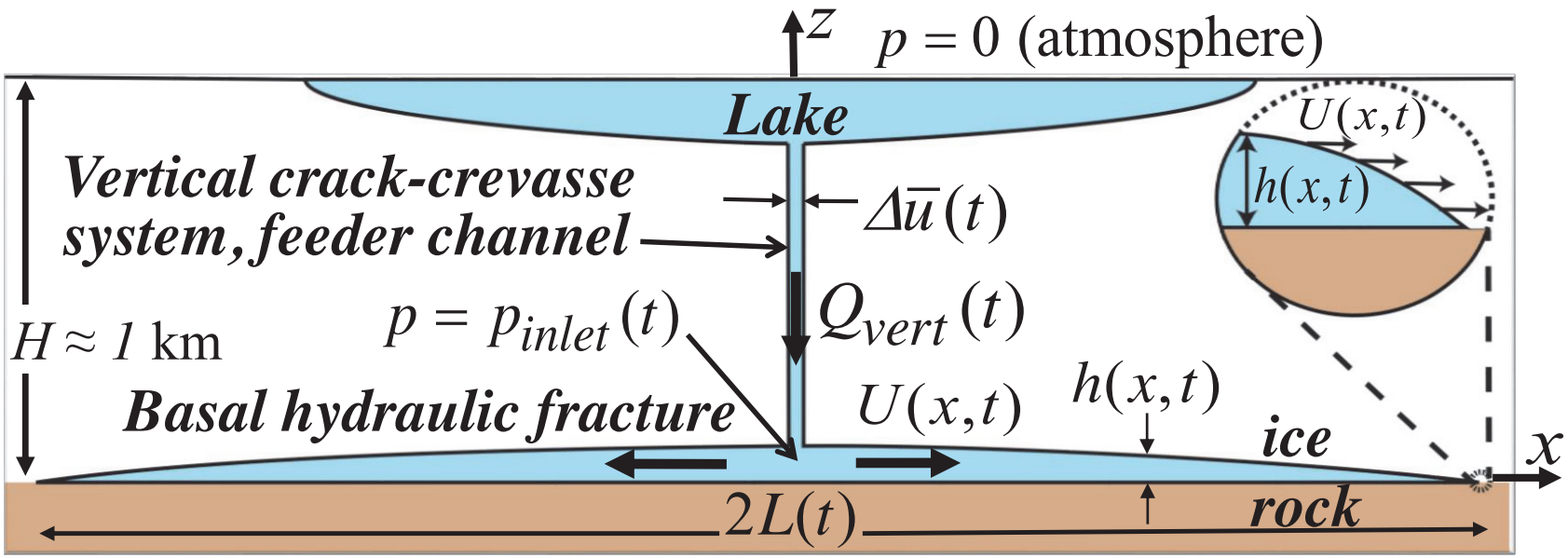
Sketch based on *Das et al. (2008)*
from *J.R. Rice, V.C. Tsai, M.C. Fernandes, J.D. Platt (2015)*

Observations of lake drainage from Das et al. (2008)



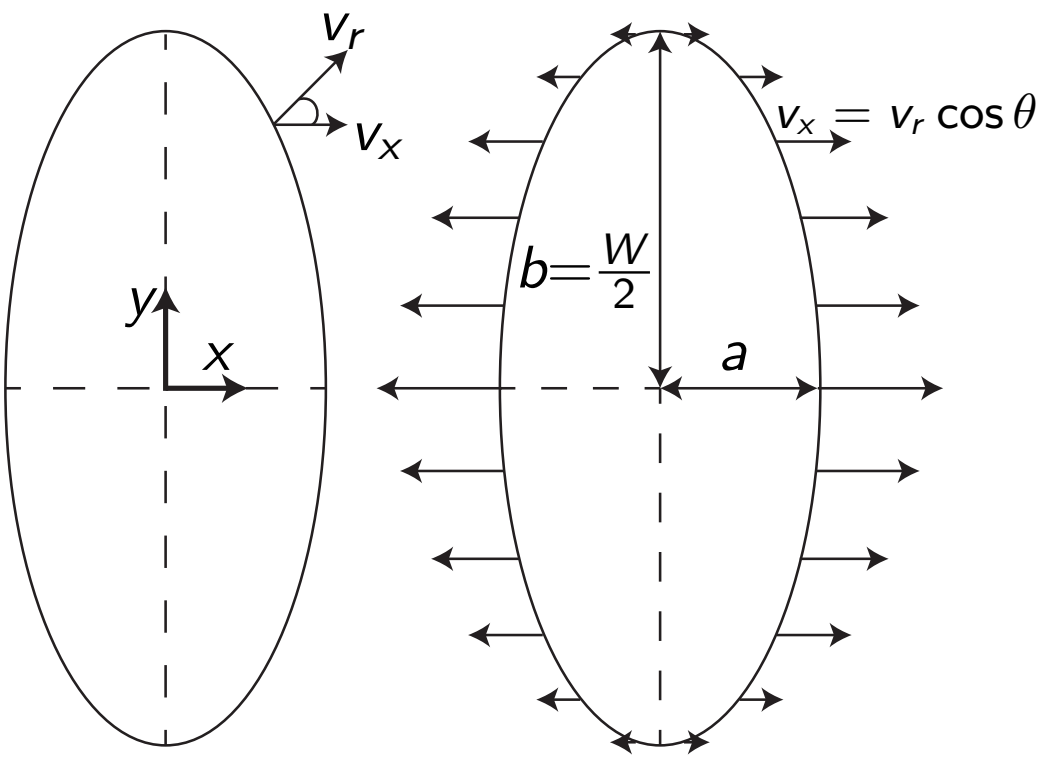
Das et al. (2008)

Schematic of crack crevasse



J.R. Rice, V.C. Tsai, M.C. Fernandes, J.D. Platt (2015)

Creep Opening of Channel



- Nye solution for radial opening of circular channel with rheological creep softness A

$$v_r = Ab\left(\frac{(p_{inlet}-\sigma_o)/2}{n}\right)^n.$$

- Were p_{inlet} is fluid pressure, and $\sigma_o = \rho_{ice} g H$ is the overburden stress, both evaluated at base of the crevasse.
- $(p_{inlet} - \sigma_o)/2$ is the average pressure through the ice sheet thickness.
- At the crack limit, the vertical component v_y will be 0.

- We can replace $\cos\theta = \sqrt{b^2 - y^2} = \sqrt{\left(\frac{W}{2}\right)^2 - y^2}$.

Model description

- Treat the vertical crevasse feeder channel as a crack.
- Assume the horizontal crack along the till has a fracture toughness $K_{Ic} = 0$, as the ice is just being lifted from the till.
- We look at the vertical crevasse average opening $\Delta\bar{u}$ as having an average elastic plus a creep part, namely

$$\Delta\bar{u} = \Delta\bar{u}^{el} + \Delta\bar{u}^{cr}.$$

- The elastic average opening is given by

$$\Delta\bar{u}^{el} = \frac{\pi}{2}\frac{(p_{inlet} - \sigma_o)/2}{E'}W,$$

where $\sigma_o = \rho_{ice}gH$ is the ice overburden stress, $(p_{inlet} - \sigma_o)/2$ is the average pressure through the ice sheet thickness and $E' = E/(1-\nu^2)$.

- We want to understand how $\Delta\bar{u}^{cr}$ plays a role.

Analytic solution for $n = 1$ and circular shape with arbitrary n

- Using a combination of Nye (1953) and Eshelby (1957) we create an expression for v^{cr}, valid for any ellipsoidal shape when $n = 1$ and valid for a circular shape when $n \neq 1$

$$v^{cr}(y) = A\left(\frac{\rho_w g H - \sigma_o}{2n}\right)^n \sqrt{\left(\frac{W}{2}\right)^2 - y^2}.$$

- We can derive the average opening rate by integrating $v^{cr}(y)dy$ over W

$$\bar{v}^{cr} = \frac{\pi}{2} A\left(\frac{\rho_w g H - \sigma_o}{2n}\right)^n W.$$

- In order to improve that simple approximation a correction factor $\kappa(n)$, which is obtained numerically, such that

$$\bar{v}^{cr} = \frac{\pi}{2} A\kappa(n)\left(\frac{\rho_w g H - \sigma_o}{2n}\right)^n W.$$

- Numerical Finite Element Method results yield $\kappa(n = 3) \approx 0.8$.

Creep contribution and elastic contribution to opening

- We define C as the ratio of the prior creep opening $\Delta\bar{u}^{cr}$ over the period of slow drainage to the elastic opening $\Delta\bar{u}^{el}$

$$C \equiv \frac{\Delta\bar{u}^{cr}\big|_{p_{inlet}=\rho_w gH}^{t=16hrs}}{\Delta\bar{u}^{el}\big|_{p_{inlet}=\rho_w gH}} = \frac{16\text{hrs} \times \bar{v}^{cr}}{\Delta\bar{u}^{el}}.$$

- Recognizing that the creep opening does not change appreciable during the short time scale of lake drainage we look at the total average opening

$$\Delta\bar{u} = \frac{\pi(p_{inlet} - \sigma_o)}{4E'}W + C\frac{\pi(\rho_w gH - \sigma_o)}{4E'}W.$$

- Parameters used for the analysis are to match those listed in Das et al. (2008), and Tsai and Rice (2010).

Parameter	H	W	ρ_w	ρ_{ice}	g	k	E'	n
Value	1	3	1000	910	9.81	0.01	6.8	3
Unit	km	km	kg/m^3	kg/m^3	m/s^2	m	GPa	-

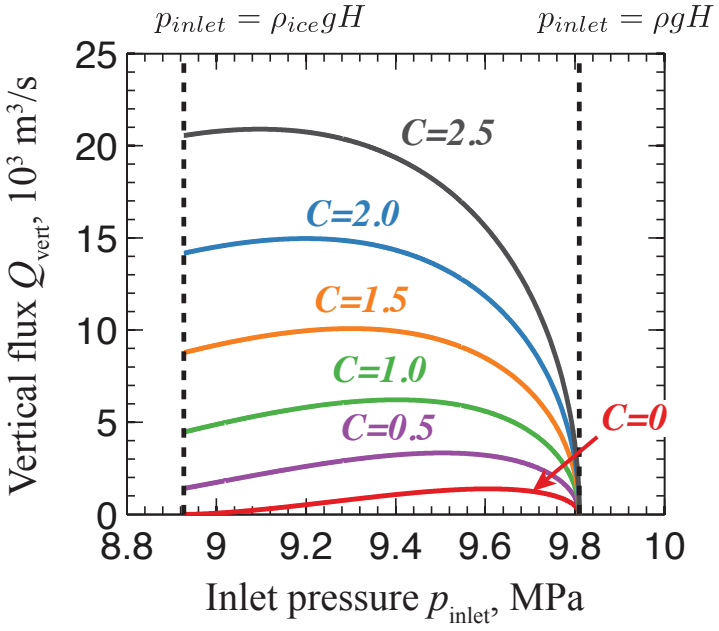
Vertical influx of lake from basal inlet pressure

- The vertical influx obtained using elementary force balance between slab weight and wall shear resistance is

$$Q_{vert} \approx 5.29 \left(1 - \frac{p_{inlet}}{\rho_w g H}\right)^{1/2} W \Delta \bar{u}^{3/2} g^{1/2} \left(\frac{\Delta \bar{u}}{k}\right)^{1/6}$$

- Using conservation of mass we can couple basal flux to vertical flux

$$Q_{basal} = Q_{vert}$$

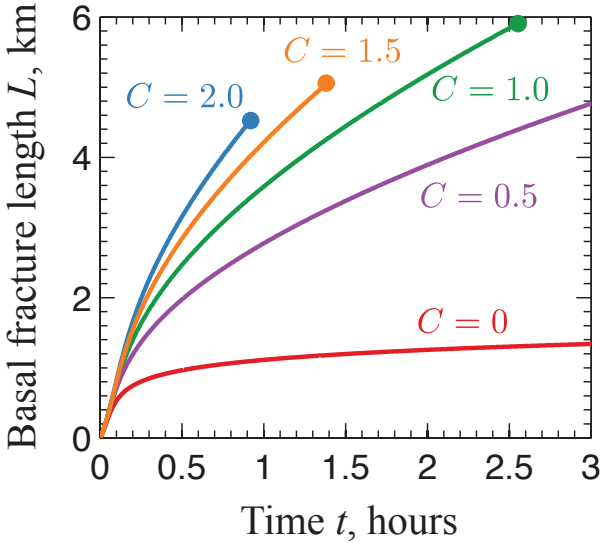


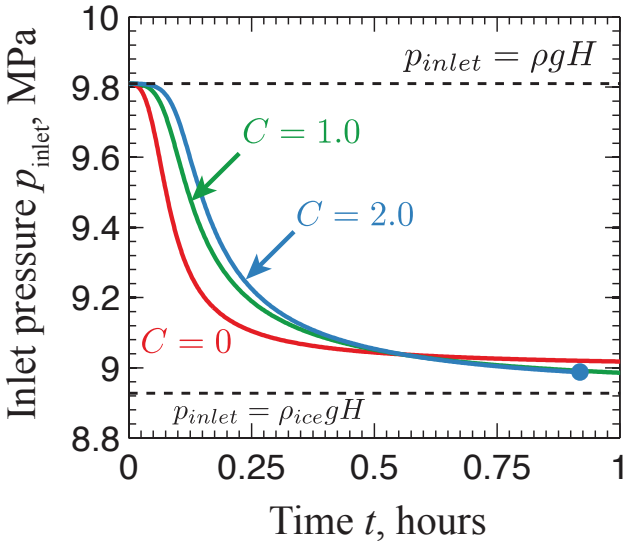
Rice et al. (2015)

- Where we can get expressions for the basal fracture propagations through the volumetric inflow rate as (Tsai and Rice, 2012)

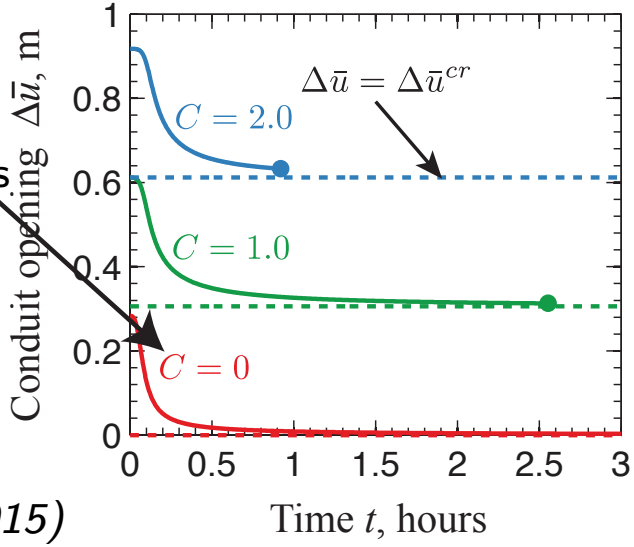
$$Q_{basal} = 6.88 \frac{(p_{inlet} - \sigma_o)}{E'} WL \left(1 + 1.034 \frac{L^2}{H^2}\right) \frac{dL}{dt}$$

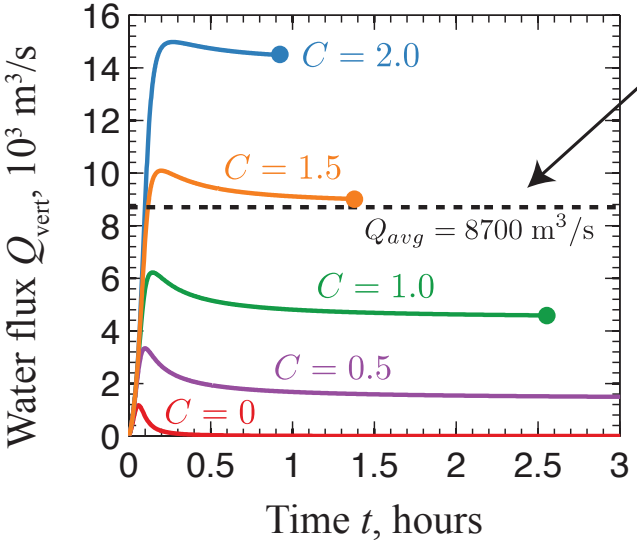
Modeling other variables as function of time





Drainage stops





Suggests $C = 1.5$

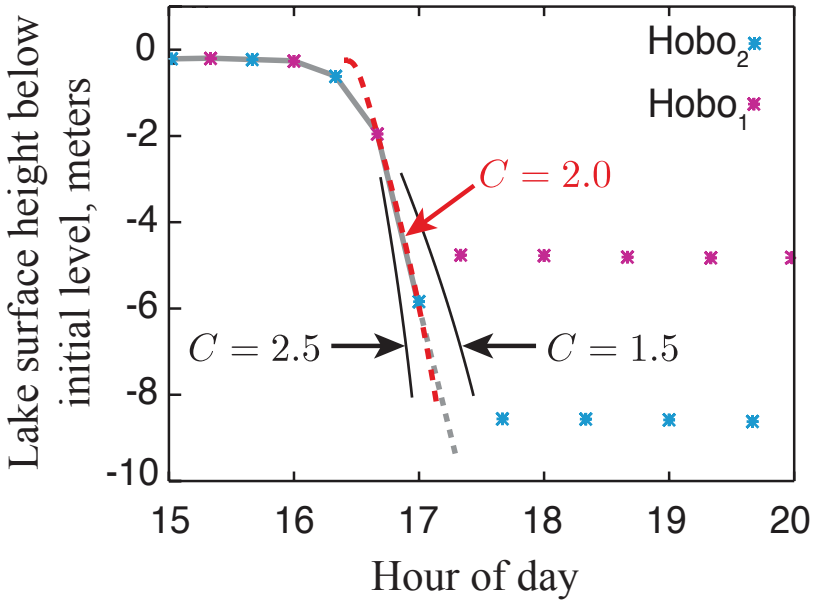
Rice et al. (2015)

Using observational evidence to get temperature of ice

- Evaluating C for the observed slow leakage period of ~ 16 hrs under hydrostatic pressurization ($p_{inlet} = \rho_w g H$)

$$C = \frac{\Delta \bar{u}^{cr}\big|^{t=16hrs}_{p_{inlet}=\rho_w g H}}{\Delta \bar{u}^{el}\big|_{p_{inlet}=\rho_w g H}}.$$

- By fitting lake surface height data over time from Das et al. (2008) we obtain that $C = 2$



Rice et al. (2015)

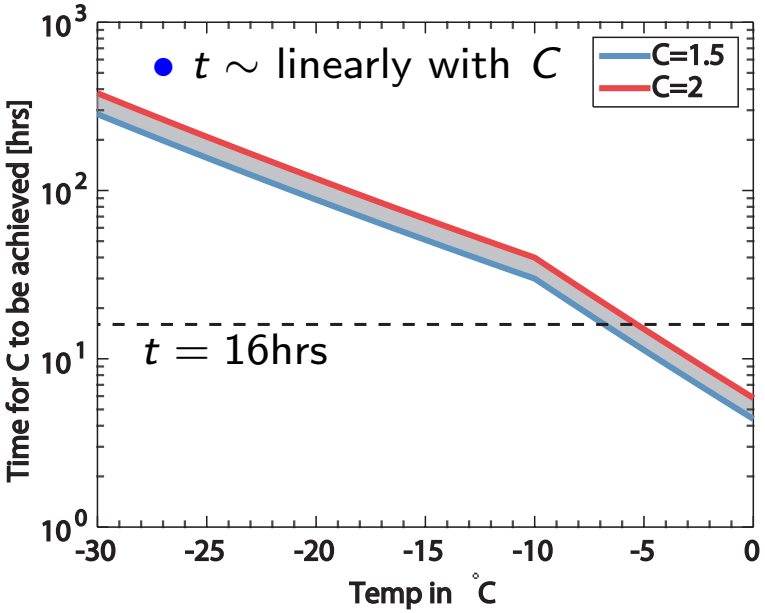
- Using data on temperature dependence our model implies that for $C = 1.5$ and $C = 2$ the ice behaves as having an average temperature of $T \approx -7.0$ °C and $T \approx -5.0$ °C, respectively.

Time to achieve C under supraglacial lakes

- $\Delta\bar{u}$ = short-time **elastic**$(\Delta\bar{u}^{el})$ + longer term **creep**$(\Delta\bar{u}^{cr})$
- $\Delta\bar{u}^{el} = \frac{\pi(p_{inlet}-\sigma_o)}{2E'}W$
- $\frac{d\Delta\bar{u}^{cr}}{dt} \approx \frac{\pi}{2}A(T)\kappa(3)\left(\frac{(\rho_w-\rho_{ice})gH}{6}\right)^3 W$ where $\kappa(3) \approx 0.8$
- $C \equiv \frac{\Delta\bar{u}^{cr}}{\Delta\bar{u}^{el}}$

Temp in °C	**A in Pa^{-3}s^{-1}**	**Time in Hrs for $C = 1$**
0	2.4e-24	~ 3
-2	1.7e-24	~ 4
-5	9.3e-25	~ 8
-10	3.5e-25	~ 20
-15	2.1e-25	~ 34
-20	1.2e-25	~ 59
-25	6.8e-26	~ 104
-30	3.7e-26	~ 190

Table values are for $C = 1$



Values of A obtained from Cuffey and Patterson (2010)

Conclusion

1. We can describe the creep opening of the vertical crevasse with an approximation using Nye and Eshelby's theories and a numerical correction $\kappa(n)$

$$\bar{v}^{cr} = \frac{\pi}{2} A\kappa(n) \left(\frac{p_{inlet} - \sigma_o}{2n} \right)^n W.$$

2. From numerical Finite Element Method simulations we obtain $\kappa(n = 3) \sim 0.8$.
3. Prior creep deformation of the crevasse is an essential component in allowing the lake to fully drain.
4. By accounting for the creep opening of the crevasse we can better estimate the vertical flux and ultimately obtain time scales and lengths for basal fracture, inlet pressure and conduit opening.
5. Using data on temperature dependence our model implies that for $1.5 < C < 2$ the ice sheet behaves as having an average temperature in the range of $-7.0\ ^\circ$C to $-5.0\ ^\circ$C.
6. Although our study focuses on the events in Das et al. (2008), the model is easily adaptable to data from other lake drainage events by simply adjusting the parameters.

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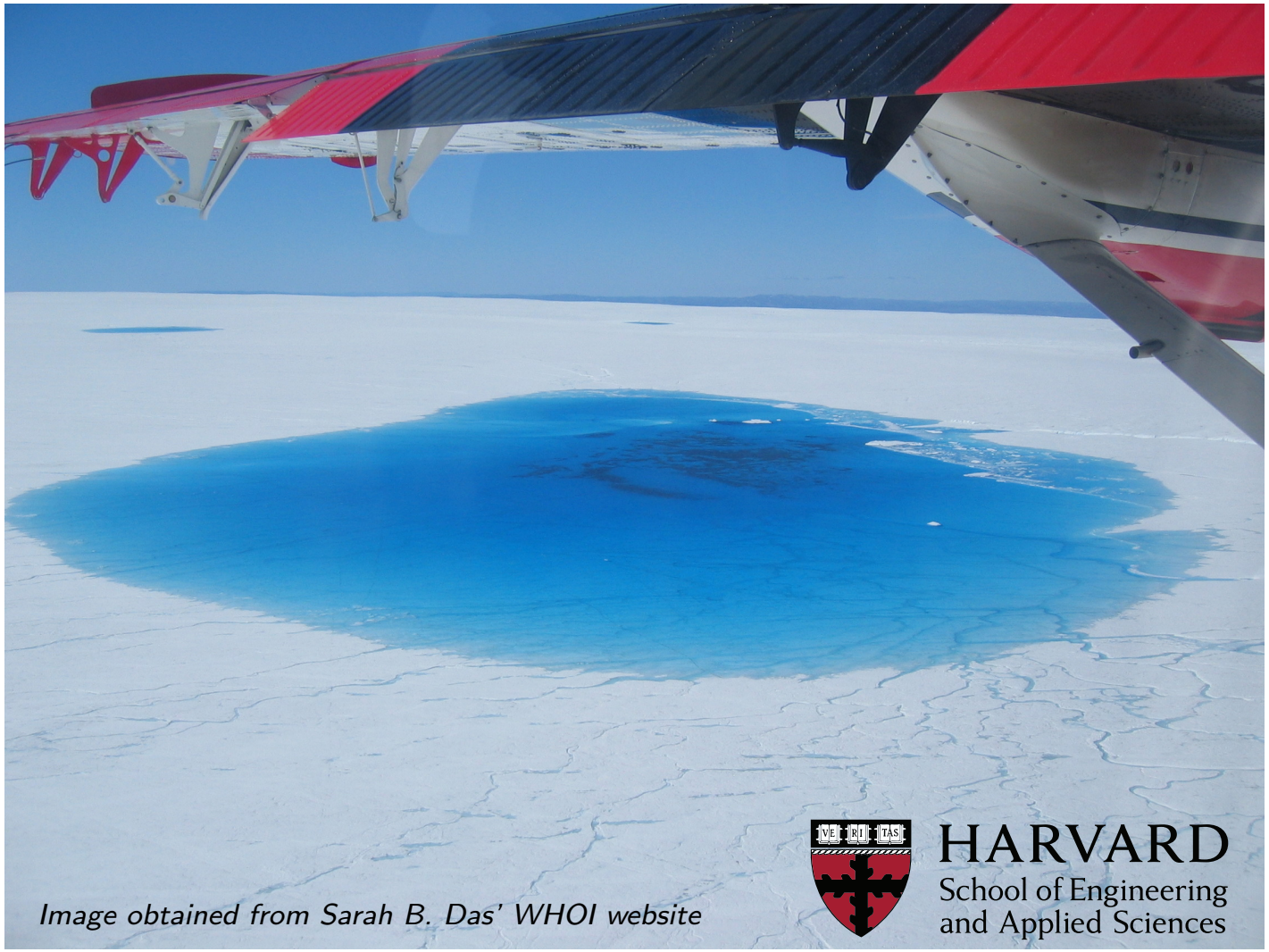
Thank You!



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