Röthlisberger Channel Model with Anti-Plane Shear Loading Superposed on In-Plane Compression

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Intro/Abstract

- The Röthlisberger channel (R-Channel) is a commonly adopted subglacial two-dimensional (2D) model described by: $L_f \rho_{ice} (\pi D/2) u_{creep} = \rho_w g \sin(\alpha) Q_w$
- R-Channel describes the balance between creep closure by in-plane straining, due to the ice overburden pressure, and the melt rate from viscous energy dissipation.
- It is estimated that melt generated from temperate ice in rapidly straining West Antarctic ice stream shear margin leads to formation of R-Channels undergoing anti-plane shear. [Perol and Rice, 2011]
- Findings by Vogel suggest that the high shear rates generated in ice stream shear margins lead to large zones of temperate ice, beneath which a Röthlisberger channel potentially forms. [Vogel et al., 2005]
- **We attempt to answer the following question:** ***For a subglacial conduit incised entirely in ice, how does the Röthlisberger channel size change under superposed antiplane shear?***

(a) Mountain Glaciers **(b)** Ice Stream Shear Margins

Figure 1: Where do we find R-Channels with Anti-Plane Shear?

Ice Rheology

- Ice is a power law creep material where strain-rate stress relationship is:

  $\dot{\epsilon}_E = A\tau_E^n$ where $\dot{\epsilon}_E = \sqrt{\frac{1}{2}\dot{\epsilon}_{ij}\dot{\epsilon}_{ij}}$ and $\tau_E = \sqrt{\frac{1}{2}S_{ij}S_{ij}}$
- Where we use Glen's flow law n=3.
- Incompressible flow: $\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy} + \dot{\epsilon}_{zz} = 0$
- The stress deviatoric tensor (S_{ij}) is: $S_{ij} = \sigma_{ij} - \delta_{ij}\frac{\sigma_{kk}}{3}$
- We assume a Plane Strain model.

Nondimensionalization (see Fig.3)

$\sigma_{rr} = N\Sigma_{rr}, \quad \dot{\epsilon}_{rr} = AN^n\dot{\mathcal{E}}_{rr}, \quad u_z = \dot{\gamma}_{far} a U_z, \quad$ and $\quad r = aR.$

The effective stress and effective strain are given as

$$\mathcal{T}_E = \sqrt{\frac{1}{4}\left(\Sigma_{rr} - \Sigma_{\theta\theta}\right)^2 + S^2\left(\Sigma_{rz}^2 + \Sigma_{\theta z}^2\right)},$$

$$\dot{\mathcal{E}}_E = \sqrt{\dot{\mathcal{E}}_{rr}^2 + S^2\left(\dot{\mathcal{E}}_{rz}^2 + \dot{\mathcal{E}}_{\theta z}^2\right)},$$

Leading to the following parameter measuring ratio of anti- to in-plane strain-rate:

$$\boxed{S = \frac{\dot{\gamma}_{far}}{AN^n}} \text{ where } N = p_{ext} - p_{chan}$$

Perturbation Solution

- Small S ($S << 1$): Pure Nye solution is dominant and the non-dimensional radial opening rate at channel is,

  $$U_r = \frac{n^{-n}\left(\frac{b}{a}\right)^2}{R\left(\left(\frac{b}{a}\right)^{\frac{2}{n}} - 1\right)^n}$$

  $U_x = C(R, a, b)\cos(\theta)$ where C is a known function
- Large S ($S >> 1$): Pure anti-plane shear solution is dominant and imposed pressure in channel is negligible.

The perturbation solution provides a numerical benchmark for the small S regime.

Geometry and Boundary Conditions

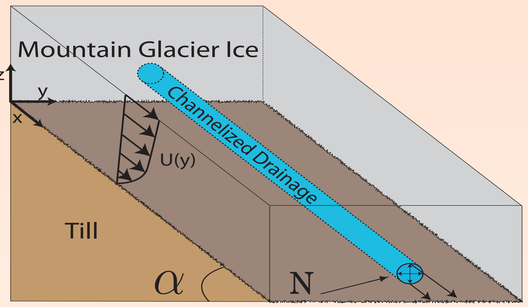

Figure 2: Mountain Glacier Anti-Plane Shear Strain

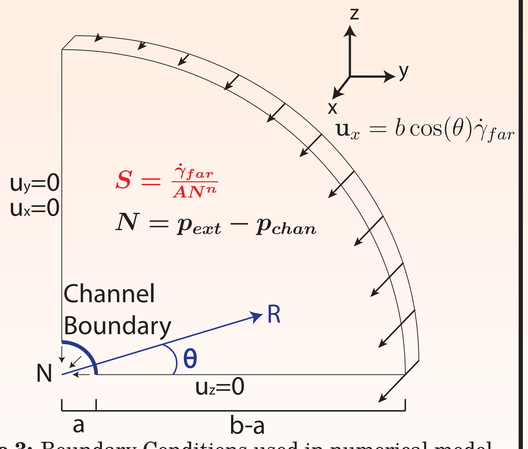

Figure 3: Boundary Conditions used in numerical model.

The boundary conditions of model consist of anti-plane shear $\dot{\gamma}_{far}$ on finite edge of domain, and a in-plane pressure is imposed on along the channel boundary.

Results

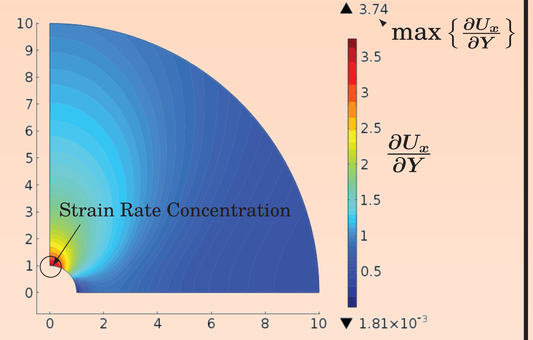

Figure 4: Numerical results for anti-plane horizontal strain rate portraying location of strain rate concentration.

- Strain concentration factor is defined as: $K_\epsilon = \max\left\{\frac{\partial U_x}{\partial Y}\right\}$

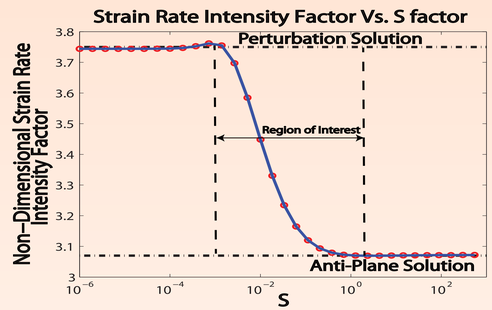

Figure 5: Numerical results closely matches (Error $\sim 0.3\%$) analytical solution at perturbation regime.

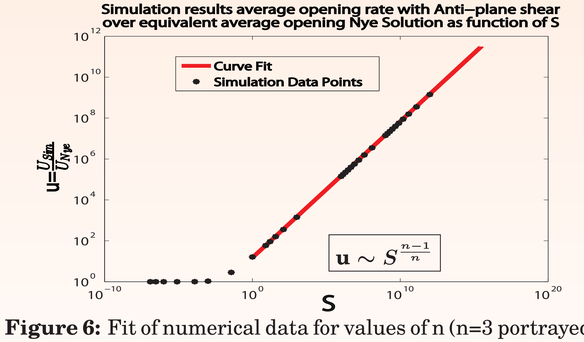

Figure 6: Fit of numerical data for values of n (n=3 portrayed in this figure), shows that at the full anti-plane regime average **u** scales as the inverse of the effective viscosity.

These plots validate the model, show the range of S where Nye solution holds, and scaling the average opening velocity from S for full anti-plane regime.

Discussion

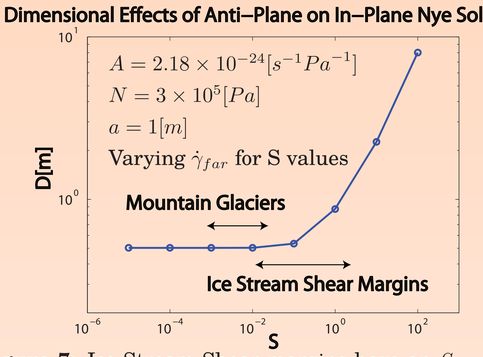

Figure 7: Ice Stream Shear margins have an $S \sim 0.01 - 1$ [Perol and Rice, 2011], and Mountain Glaciers have an $S \sim 0.001 - 0.05$ [Bartholomaus et al., 2011].

Ice-stream shear margins have anti-plane strains that effect 2D Nye Solution, whereas the effects in Mountain Glaciers will be negligible.

Conclusion

1. The contributions of anti-plane shear will impact the opening of the channel from the pure in-plane Röthlisberger solution for values of S found in Mountain Glaciers and Ice Sheet Shear Margins.
2. In large S regime, where the anti-plane components are dominant, the ratio of the anti-plane to pure in-plane Röthlisberger solution scales as $u \sim S^{\frac{n-1}{n}}$, which is the inverse of the effective viscosity.

References

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Acknowledgements

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